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A variable flavour number scheme for charged current heavy flavour structure functions

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Abstract. The Thorne-Roberts variable flavour number scheme (VFNS) for heavy quarks is presented in detail for the specific case of charged current DIS. As in neutral current DIS this provides a smooth extrapolation from the fixed flavour number scheme (FFNS) appropriate at low Q^2 to the zero-mass variable flavour number scheme (ZM-VFNS) appropriate as $Q^2 \to \infty$, and differs from alternative versions of a VFNS by the definition of the coefficient functions at each order, and the strict ordering of the expansion in α_S. However, there are subtle differences from the neutral current case which are addressed here. We discuss both the LO and NLO expressions, the latter unfortunately requiring some (minimal) modelling due to the current lack of some necessary $\mathcal{O}(\alpha_S^2)$ FFNS coefficient functions.

1 Introduction

In the past few years direct measurements of charm production at HERA [1,2], as well as the fact that the charm structure function F_2^c can be 20% or more of the total F_2, have made a consistent theoretical framework for heavy flavour production in neutral current deep inelastic scattering (DIS) essential. For $Q^2 \lesssim m_c^2$, where m_c is the charm quark mass, the conventional description in terms of order-by-order in α_s coefficient functions for the production of charm in the final state is perfectly satisfactory, but for $Q^2 >> m_c^2$ this description becomes potentially unreliable due to the presence of logarithms in $\ln(Q^2/m_c^2)$ at all orders in α_s which ideally should be resummed. By changing to the alternative description where the charm quark is treated as a parton this resummation is automatically performed and, at the same time, a complete set of parton densities needed to calculate other processes involving nucleons is obtained. It is relatively straightforward to do this by treating charm as a massless parton, thus obtaining the correct high Q^2 limit, but more challenging to obtain a treatment which successfully includes the charm mass effects for Q^2 not too far above m_c^2.

This problem was first addressed in [3], and the term variable flavour number scheme (VFNS) coined for a general order-by-order prescription for the calculation of F_2^c which extrapolates from $Q^2 \leq m_c^2$, to the asymptotic limit $Q^2/m_c^2 \to \infty$. However, while the prescription in [3] is certainly formally correct (an all orders proof being presented in [4])[1] it is arguable that it is not the most efficient and elegant definition of a VFNS. In particular, the correct threshold behaviour for charm pair production is not precisely maintained order by order, leading to a lack of smoothness when one begins charm parton evolution, particularly in F_L^c. In [6] we developed a VFNS which while to all orders is identical to that in [3][2], differs at any fixed order in perturbation theory, both due to the definition of the coefficient functions and due to the way in which we define what a given order actually means. Essentially, we use the inherent freedom in the definition of the coefficient functions corresponding to charm partons to ensure smoothness of the structure functions across the transition point where we switch from a three to a four flavour scheme. This results in the "Thorne-Roberts" VFNS.

Recently it has become apparent that the treatment of heavy flavours is also very important in the context of charged current scattering. In particular, a consistent method is needed in order to explain the data obtained by CCFR [8], and resolves the long standing discrepancy between this and the NMC muon data [9] for $x \leq 0.01$ (see [10] for a presentation of the "Physics-Model Independent" treatment of this data, and [11] for a discussion of the theoretical issues involved). Although we outlined the way in which one treats charged currents in the TR

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[1] Strictly speaking this scheme, as well as ours, applies to the total structure function F_2 - the explicit separation of a charm component F_2^c becoming ambiguous beyond NLO, as discussed in [5]

[2] This is only exactly true if there is no intrinsic charm. If there is intrinsic charm the schemes differ by $\mathcal{O}(\Lambda^2_{\rm QCD}/Q^2)$ [7], i.e. of the order of the error in perturbative QCD

scheme in the latter of [6], and produced publicly available code at the time, closer examination has revealed that the issue is more subtle than we originally believed, and that the original code contained some errors. Hence, in this paper we will present the explicit form of the TR VFNS for charged currents, and accompany the paper with a revised code. The paper contains our results at leading order (LO) and next-to-leading order (NLO). The former of these is complete, while unfortunately the latter contains a certain amount of modelling since the NLO fixed flavour number scheme (FFNS) coefficient functions, which would be necessary in the full prescription, have not yet been calculated. We find that a minimal degree of sophistication is needed in the modelling however.

2 Definition of the VFNS at LO

The general formulation of the VFNS for charged current interactions was presented in the appendix of the former of [6]. Here we shall be more explicit, and work order by order. We consider the process $W^+ \to c, \bar{s}$ or $W^+ \to c, \bar{d}$. The whole of the variable flavour number scheme is based on the idea that ignoring any intrinsic heavy flavour (as we shall do in this paper) the partons in the 4-flavour scheme are related to those in the 3-flavour scheme by

$$f_b^4(z,\mu^2,m_c^2/\mu^2) = A^{ba}(\mu^2/m_c^2) \otimes f_a^3(\mu^2), \qquad (1)$$

where the $A^{ba}(\mu^2/m_c^2)$ are perturbatively calculable matrix elements which are known to $\mathcal{O}(\alpha_S^2)$ [12]. By using the exact equivalence of the total structure function calculated in either the FFNS or the VFNS, i.e.

$$\begin{aligned} F_i(x,Q^2,m_c^2) &= C_a^{FF} \otimes f_a^3(\mu^2) \\ &\equiv C_b^{VF} \otimes f_b^4(z,\mu^2,m_c^2/\mu^2), \end{aligned} \qquad (2)$$

and using (1) one obtains the VFNS coefficient functions in the implicit form

$$\begin{aligned} &C_b^{VF}(z,Q^2/\mu^2,m_c^2/\mu^2) \\ &\quad = C_a^{FF}(Q^2/\mu^2,m_c^2/\mu^2) \otimes \left[A^{ba}(\mu^2/m_c^2)\right]^{-1}. \end{aligned} \qquad (3)$$

Since the index b runs over one more value than index a the above equation does not have unique solutions, and there is an inherent freedom in the definition of the VFNS coefficient functions to which the parton distributions are completely insensitive. In [3] they were calculated explicitly in $\overline{\rm MS}$ scheme using charm quarks in the initial state, and all collinear divergences systematically removed. The same direct definition of coefficient functions is used in [5]. However, at fixed order this definition can lead to single charm quark production below the real threshold for pair production, as well as a lack of smoothness when switching from 3 to 4 flavours (depending on renormalization/factorization scale), as clearly seen in Fig. 18 of [5]. Hence, we use an alternative method to define the coefficient functions while using the same parton distributions. We use the freedom to redefine coefficient functions while still satisfying (3) and solve (3) order by order, removing the freedom by imposing the continuity of $dF_i(x,Q^2)/d\ln Q^2$ (in the gluon sector) across the transition point. This then guarantees the correct threshold behaviour in each coefficient function, and smoothness when switching flavour number. The difference between this scheme and [3] and [5] is effectively a change of factorization scheme such that coefficient functions differ by $\mathcal{O}(m_c^2/Q^2)$, but where the parton distributions are identical in the two schemes. Hence, our VFNS uses the standard 4-flavour $\overline{\rm MS}$ partons.

The case of charged current scattering is made more complicated than that for neutral currents by the fact that the heavy quark is often produced along with, or from, a light quark, rather than in a heavy quark-antiquark pair. Hence, within the FFNS it is not the production of the charm quark which vanishes at zeroth order in α_S. In fact at LO the charm quark structure function is given by

$$\begin{aligned} F_2^{c,LO}(x) &= 2\,[\,\cos^2\theta_c\;\; \xi s(\xi) + \sin^2\theta_c\;\; \xi d(\xi)\,] \\ xF_3^{c,LO}(x) &= 2\,[\,\cos^2\theta_c\;\; x s(\xi) + \sin^2\theta_c\;\; x d(\xi)\,] \end{aligned} \qquad (4)$$

where $\xi = x/x_0$, $x_0 = 1/(1+\epsilon)$ and $\epsilon = m_c^2/Q^2$. The partons being functions of ξ rather than x due to the need to put the charm quark on mass-shell. From now on we will denote $[\,\cos^2\theta_c\;\; \xi s + \sin^2\theta_c\;\; \xi d\,]$ by $\tilde{s}$. At zeroth order it is the production of the weak eigenstate conjugate to c, i.e. $\bar{\tilde{s}}$, which has zero production cross-section in the FFNS. In this scheme the LO contribution to $\bar{\tilde{s}}$ production is (we choose $\mu^2 = Q^2$)

$$F_i^{\bar{\tilde{s}}}(x) = 2\left(\frac{\alpha_S}{4\pi}\right)\int_x^{x_0} dz\; C_{i,g}^{(1)FF}(z,\epsilon)\;\tilde{g}(x/z) \qquad (5)$$

where $\tilde{g}(x) = xg(x)$, and in this paper we consider the cases $i = 2$ and $i = 3$. The fixed flavour coefficient functions on the rhs of (5) are related to the the $W^+ g \to c\bar{s}$ coefficients given by [13]

$$\begin{aligned} C_{2,g}^{(1)FF}(z,\epsilon) &= \frac{2}{x_0}\; H_2^g(\frac{z}{x_0}) \\ C_{3,g}^{(1)FF}(z,\epsilon) &= 2\; H_3^g(\frac{z}{x_0}), \end{aligned} \qquad (6)$$

which are an update of those in [14] to account for the correct counting of gluon helicity states in $D = 4 + 2\epsilon$ dimensions. The factor of 2 is just our convention while the factors of x_0 come from a change of variables in the integration defining the convolution compared to [13].

Above the transition point[3], which as before we choose for convenience to be $Q^2 = m_c^2$, one can produce $\bar{\tilde{s}}$ quarks directly from initial state charm quarks, i.e. at LO the (Q^2-dependent) LO expression is

$$F_i^{\bar{\tilde{s}}}(x) = \int_x^{x_0} dz\; C_{i,\bar{\tilde{s}}c}^{(0)VF}(z,\epsilon)\;\tilde{c}(x/z). \qquad (7)$$

[3] Throughout we ignore reference to the number of flavours concerning α_S. However, $\alpha_S(Q^2)$ does change across the transition point as discussed for the neutral current case in [6]

In principle we now impose the continuity of $\frac{dF_i^{\bar{s}}(x)}{d\ln Q^2}$ at first order in α_S to obtain

$$\frac{dC_{i,g}^{(1)FF}(z,\epsilon)}{d\ln Q^2} = C_{i,\bar{s}c}^{(0)VF}(\epsilon) \otimes P_{qg}^{(0)}, \tag{8}$$

where $P_{qg}^{(0)}(z) = 1/2(z^2+(1-z)^2)$, i.e. the LO quark-gluon splitting function. As shown in [6] one may invert (8), more easily by considering the ultimate convolution with the charm density, obtaining

$$\begin{aligned} &C_{i,\bar{s}c}^{(0)\,VF}(\epsilon) \otimes c(Q^2) \\ &= -\int_x^{x_0} dz\, \frac{dC_{i,g}^{(1)FF}(z,\epsilon)}{d\ln Q^2}\left(\frac{x}{z}\right)^2 \frac{dc(x/z,Q^2)}{d(x/z)} \\ &\quad +2\int_x^{x_0} dz\, \frac{dC_{i,g}^{(1)FF}(z,\epsilon)}{d\ln Q^2}\,\frac{x}{z}\,c(x/z,Q^2) \\ &\quad -2\int_x^{x_0} dz\, \frac{dC_{i,g}^{(1)FF}(z,\epsilon)}{d\ln Q^2} \\ &\quad \times \int_{x/z}^1 dz'\, r(z')\,\frac{x}{zz'}\,c(x/zz',Q^2), \end{aligned} \tag{9}$$

where $r(z)$ is given by

$$r(z) = z^{\frac{1}{2}}\left[\cos\Big(\frac{\sqrt{7}}{2}\ln\frac{1}{z}\Big) + \frac{3}{\sqrt{7}}\sin\Big(\frac{\sqrt{7}}{2}\ln\frac{1}{z}\Big)\right]. \tag{10}$$

However, the fact that in the charged current case the boson-gluon fusion process leads to a charm quark plus a light quark, rather than the charm quark-antiquark pair of the neutral current, leads to a technical complication. For the neutral current the LO boson-gluon fusion coefficient function is infrared finite, and since it corresponds to the production of two massive quarks, vanishes smoothly at the kinematic threshold of $\hat{W}^2 = 4m_c^2$, where $\hat{W}^2 = Q^2(1/z-1)$. In contrast the charged-current boson-gluon fusion has a collinear divergence due to the final state light quark which must be regularized using dimensional regularization, and a subtraction made according to the rules of collinear factorization. The remaining finite coefficient function is no longer a real cross-section, and although it vanishes for $\hat{W}^2$ below the threshold of $\hat{W}^2 = m_c^2$, it does not tend to zero at the threshold, and in fact is logarithmically divergent as $\hat{W}^2 \to m_c^2$. The non-vanishing means that when taking the derivative of the right-hand side of (5) with respect to $\ln Q^2$ one must also take account of the end-point of the derivative, i.e. we actually replace (8) by

$$\begin{aligned} &\frac{d}{d\ln Q^2}(C_{i,g}^{(1)FF}(\epsilon) \otimes \tilde{g}(Q^2)) \\ &\quad = C_{i,\bar{s}c}^{(0)VF}(\epsilon) \otimes P_{qg}^{(0)} \otimes \tilde{g}(Q^2), \end{aligned} \tag{11}$$

The fact that the coefficient function is divergent at this point means the end-point contribution must be treated with particular care.

Thus, in order to define $C_{i,\bar{s}c}^{(0)VF}(z,\epsilon)$ we separate out the part of the gluon coefficient function which diverges as $z \to x_0$ by writing

$$C_{i,g}^{(1)FF}(z,\epsilon) = C_{i,g,reg.}^{(1)FF}(z,\epsilon) + C_{i,g,dvgt.}^{(1)FF.}(z,\epsilon).$$

This results in

$$\begin{aligned} &C_{2,g,reg.}^{(1)FF}(z,\epsilon) \\ &= \frac{1}{x_0}\left\{2P_{qg}^{(0)}\left(\frac{z}{x_0}\right)\left[L_\lambda\left(\frac{z}{x_0}\right) - \ln x_0\right]\right\} \\ &\quad +2[8-18\,(1-x_0)+12\,(1-x_0)^2]\frac{z}{x_0}\left(1-\frac{z}{x_0}\right) \\ &\quad +2\left[\frac{1-x_0}{1-z}-1\right] + 2P_{qg}^{(0)}\left(\frac{z}{x_0}\right)\ln\left[\frac{x_0}{(1-z)z}\right] \\ &\quad \times\left\{+12(1-x_0)z(1-2z)L_\lambda\left(\frac{z}{x_0}\right)\right\}, \end{aligned} \tag{12}$$

where $L_\lambda(z) = \ln\left[\frac{x_0(1-z)}{(1-x_0)z}\right]$, and

$$C_{2,g,dvgt.}^{(1)FF}(z,\epsilon) = \frac{4}{x_0}P_{qg}^{(0)}\left(\frac{z}{x_0}\right)\ln\left(1-\frac{z}{x_0}\right). \tag{13}$$

Also

$$\begin{aligned} &C_{3,g,reg.}^{(1)FF}(z,\epsilon) \\ &= 2P_{qg}^{(0)}\left(\frac{z}{x_0}\right)\left[-L_\lambda\left(\frac{z}{x_0}\right) - \ln x_0\right] + 4(1-x_0)\frac{z}{x_0} \\ &\quad \times\left(1-\frac{z}{x_0}\right) + 2(1-x_0)\frac{z}{x_0}L_\lambda\left(\frac{z}{x_0}\right) \\ &\quad \times\left[-2\left(1-\frac{z}{x_0}\right)+2z\right] + 2P_{qg}^{(0)}\left(\frac{z}{x_0}\right) \\ &\quad \times\ln\left[\frac{x_0}{(1-z)z}\right] \end{aligned} \tag{14}$$

and

$$C_{3,g,dvgt.}^{(1)FF}(z,\epsilon) = x_0 C_{2,g,dvgt.}^{(1)FF}(z,\epsilon) \tag{15}$$

It is easy to check that these coefficient functions approach the appropriate limits as $Q^2/m_c^2 \to \infty$. In this limit $\epsilon \to 0$, $x_0 \to 1$ and

$$\begin{aligned} C_{2,g}^{(1)FF}(z,\epsilon) \to 2\Bigg\{&8z(1-z)-1+2P_{qg}^{(0)}(z)\ln\left[\left(\frac{1-z}{z}\right)\right] \\ &+P_{qg}^{(0)}(z)\ln(1/\epsilon)\Bigg\} \end{aligned} \tag{16}$$

and

$$C_{3,g}^{(1)FF}(z,\epsilon) \to -P_{qg}^{(0)}(z)\ln(1/\epsilon). \tag{17}$$

Therefore both coefficient functions approach the massless form plus the appropriate collinear logarithms for the absorption into charm evolution.

In the VFNS, $Q > m_c^2$, we need

$$\begin{aligned} & C_{i,\bar{s}c}^{(0)VF}(\epsilon) \otimes P_{qg}^{(0)} \otimes \tilde{g}(Q^2) \\ & = \frac{d}{d\ln Q^2}\left[C_{i,g}^{(1)FF}(\epsilon) \otimes \tilde{g}(Q^2)\right] \\ & = \frac{d}{d\ln Q^2}\left(\left[C_{i,g,reg.}^{(1)FF}(\epsilon) + C_{i,g,dvgt.}^{(1)FF}(\epsilon) \otimes \tilde{g}(Q^2)\right]\right). \end{aligned} \tag{18}$$

Let us consider each of these derivatives in turn. Strictly the derivative is of the convolution so the terms generated by differentiating the end point of the integration must be included. We first consider the regular piece.

$$(a) \qquad \frac{d}{d\ln Q^2}\left[C_{2,g,reg.}^{(1)FF}(\epsilon) \otimes \tilde{g}(Q^2)\right].$$

This results in

$$\begin{aligned} & \frac{d}{d\ln Q^2}\left[\int_x^{x_0} dz\, C_{2,g,reg.}^{(1)FF}(z,\epsilon)\, \tilde{g}(x/z)\right] \\ & = \epsilon x_0^2 C_{2,g,reg.}^{(1)FF}(x_0,\epsilon)\tilde{g}(x/x_0) \\ & \quad + \epsilon x_0^2 \int_x^{x_0} dz\, \frac{d}{dx_0}[C_{2,g,reg.}^{(1)FF}(z,\epsilon)]\, \tilde{g}(x/z) \\ & = -\epsilon x_0 \ln[x_0(1-x_0)]\tilde{g}(x/x_0) \\ & \quad + \epsilon x_0^2 \int_x^{x_0} dz\, \frac{d}{dx_0}[C_{2,g,reg.}^{(1)FF}(z,\epsilon)]\, \tilde{g}(x/z) \end{aligned} \tag{19}$$

and using (12) we get

$$\begin{aligned} & \frac{d}{dx_0}[C_{2,g,reg.}^{(1)FF}(z,\epsilon)] \\ & = \left[\frac{2}{x_0^2(1-x_0)}\right] P_{qg}^{(0)}\left(\frac{z}{x_0}\right) - \frac{2}{x_0^2(1-z)} + \frac{12z(1-2z)}{x_0^2} \\ & \quad \times \left[1 - L_\lambda\left(\frac{z}{x_0}\right)\right] + \frac{(4zx_0 - 6z^2 - x_0^2)}{x_0^4}\ln\left[\frac{x_0}{(1-z)z^2}\right] \\ & \quad + \frac{4z}{x_0^2}\left[3z - 2(1+3z)x_0 + 3(1+2z)x_0^2\right] \end{aligned} \tag{20}$$

Now we also consider the divergent piece.

$$(b) \qquad \frac{d}{d\ln Q^2}\left[C_{2,g,dvgt.}^{(1)FF}(\epsilon) \otimes \tilde{g}(Q^2)\right].$$

Differentiating this we obtain

$$\begin{aligned} & \frac{d}{d\ln Q^2}\left[\int_x^{x_0-\delta} dz\, C_{2,g,dvgt.}^{(1)FF}(z,\epsilon)\, \tilde{g}(x/z)\right] \\ & = \epsilon x_0^2 C_{2,g,dvgt.}^{(1)FF}(x_0-\delta,\epsilon)\tilde{g}(x/(x_0-\delta)) \\ & \quad + \epsilon x_0^2 \int_x^{x_0-\delta} dz\, \frac{d}{dx_0}\left[C_{2,g,dvgt.}^{(1)FF}(z,\epsilon)\right]\, \tilde{g}(x/z), \end{aligned} \tag{21}$$

where for the moment we have moved the upper limit of integration an infinitesimal amount δ below x_0. Writing

$$C_{2,g,dvgt.}^{(1)FF}(z,\epsilon) = \phi(z,\epsilon)\ln\left(1-\frac{z}{x_0}\right),$$

we get

$$\begin{aligned} & \frac{d}{d\ln Q^2}\left[\int_x^{x_0-\delta} dz C_{2,g,dvgt.}^{(1)FF}(z,\epsilon)\tilde{g}(x/z)\right] \\ & = \epsilon x_0^2 \phi(x_0,\epsilon)\tilde{g}(x/(x_0-\delta))\ln\left(1-\frac{x_0-\delta}{x_0}\right) \\ & \quad + \epsilon x_0^2 \int_z^{x_0-\delta} dz\, \frac{d}{dx_0}\left[\phi(z,\epsilon)\ln\left(1-\frac{z}{x_0}\right)\right] \\ & \quad \times \tilde{g}(x/z). \end{aligned} \tag{22}$$

Now

$$\begin{aligned} & \frac{d}{dx_0}\left[\phi(z,\epsilon)\ln\left(1-\frac{z}{x_0}\right)\right] \\ & = \frac{z}{x_0^2(1-\frac{z}{x_0})}\phi(z,\epsilon) + \frac{d\phi(z,\epsilon)}{dx_0}\ln\left(1-\frac{z}{x_0}\right) \end{aligned} \tag{23}$$

and since, in this case, $\phi(z,\epsilon) = \frac{4}{x_0}P_{qg}^{(0)}(\frac{z}{x_0})$ then

$$\begin{aligned} & \frac{d\phi(z,\epsilon)}{dx_0}\ln\left(1-\frac{z}{x_0}\right) \\ & \quad = \frac{2}{x_0^4}\left(4zx_0 - 6z^2 - x_0^2\right)\ln\left(1-\frac{z}{x_0}\right) \end{aligned}$$

and this contribution to the convolution can be added to the regular contribution given by (20). Now the first term on the rhs of (23) inserted into (22) gives

$$\begin{aligned} & \epsilon\int_x^{x_0-\delta} \frac{dz}{1-\frac{z}{x_0}}\, z\phi(z,\epsilon)\tilde{g}(x/z) \\ & = \epsilon x_0\phi(x_0,\epsilon)\tilde{g}(x/x_0)\int_x^{x_0-\delta}\frac{dz}{1-\frac{z}{x_0}} + \epsilon\int_x^{x_0-\delta}\frac{dz}{1-\frac{z}{x_0}} \\ & \quad \times [z\phi(z,\epsilon)\tilde{g}(x/z) - x_0\phi(x_0,\epsilon)\tilde{g}(x/x_0)] \\ & = -\epsilon x_0^2\phi(x_0,\epsilon)\tilde{g}(x/x_0)\ln\left(1-\frac{x_0-\delta}{x_0}\right) \\ & \quad + \epsilon x_0^2\phi(x_0,\epsilon)\tilde{g}\left(\frac{x}{x_0}\right)\ln\left(1-\frac{x}{x_0}\right) + \epsilon\int_x^{x_0-\delta} \\ & \quad \times \frac{dz}{1-\frac{z}{x_0}}[z\phi(z,\epsilon)\tilde{g}(x/z) - x_0\phi(x_0,\epsilon)\tilde{g}(x/x_0)]. \end{aligned} \tag{24}$$

The first term in (24) cancels the first term in (22) (up to $\mathcal{O}(\delta)$), and hence all divergences cancel as $\delta \to 0$. Removing these two terms and now safely setting $\delta = 0$ the second term can be added to the first term of (19) as the net 'local' contribution to the convolution.

So (a) and (b) together give the following contributions:

$$\begin{aligned} & \text{'local'term} : \epsilon x_0\left[2\ln\left(1-\frac{x}{x_0}\right) - \ln\left(x_0(1-x_0)\right)\right] \\ & \qquad \tilde{g}(\frac{x}{x_0}), \\ & \text{`}+\text{'} \text{ term} : \frac{2\epsilon}{x_0}\int_x^{x_0}\frac{dz}{1-\frac{z}{x_0}} \end{aligned} \tag{25}$$

$$\left[2zP_{qg}^{(0)}(\frac{z}{x_0})\tilde{g}(x/z) - x_0\tilde{g}\left(\frac{z}{x_0}\right)\right], \quad (26)$$

$$\text{'regular'term} : \epsilon x_0^2 \int_x^{x_0} dz \left\{ \frac{d}{dx_0} C_{2,g,reg.}^{(1)} (z,\epsilon)\Big|_{\text{(as given in (20))}} + \frac{2}{x_0^4}(4zx_0 - 6z^2 - x_0^2)\ln\left(1-\frac{z}{x_0}\right)\right\} \tilde{g}(x/z)$$

$$\equiv \int_x^{x_0} dz \left(\frac{dC_{2,g}^{(1)}(z,\epsilon)}{d\ln Q^2}\right)_{reg} \tilde{g}(x/z). \quad (27)$$

As $Q^2/m_c^2 \to \infty$, the only surviving term comes from the regular piece which $\to 2P_{qg}^{(0)}(z)$, and hence clearly using (8), $C_{2,\bar{\bar{s}}c}^{(0)VF}(z,\epsilon) \to 2z\delta(1-z)$ in this limit.

In general we can use the three contributions to $\frac{d}{d\ln Q^2} C_{2,g}^{(1)FF}(\epsilon) \otimes \tilde{g}(Q^2)$ to derive three contributions to the charm quark coefficient function $C_{\bar{\bar{s}}c}^{(0)VF}(z,\epsilon)$, convoluted with the charm density. The part coming from the 'regular' term (27) contributes in the normal manner as in (9). The part coming from the local term is even simpler, becoming

$$C_{i,\bar{\bar{s}}c}^{(0)\,VF\,\mathrm{loc}}(\epsilon) \otimes c(Q^2)$$
$$= -f_{\mathrm{loc}}(x,x_0)\left(\frac{x}{x_0}\right)^2 \frac{dc(x/x_0,Q^2)}{d(x/x_0)} + 2f_{\mathrm{loc}}(x,x_0)\,\frac{x}{x_0}\,c(x/x_0,Q^2) - \frac{2}{x_0}\int_x^{x_0} dz f_{\mathrm{loc}}(x,x_0) r\left(\frac{z}{x_0}\right)\,\frac{x}{z}\,c(x/z,Q^2), \quad (28)$$

where

$$f_{\mathrm{loc}}(x,x_0) = \epsilon x_0\left[2\ln\left(1-\frac{x}{x_0}\right) - \ln\left(x_0(1-x_0)\right)\right]. \quad (29)$$

The part coming from the '+' term is the most complicated. For the first two terms in the expression of the form (9) it is relatively straightforward, i.e. we obtain

$$-\frac{2\epsilon}{x_0}\int_x^{x_0} \frac{dz}{1-\frac{z}{x_0}}\left[2zP_{qg}^{(0)}\left(\frac{z}{x_0}\right)\left(\frac{x}{z}\right)^2\frac{dc(x/z)}{d(x/z)} - x_0\left(\frac{x}{x_0}\right)^2\frac{dc(x/x_0)}{d(x/x_0)}\right] + 2\frac{2\epsilon}{x_0}\int_x^{x_0}\frac{dz}{1-\frac{z}{x_0}} \times \left[2zP_{qg}^{(0)}\left(\frac{z}{x_0}\right)\tilde{c}(x/z) - x_0\tilde{c}(x/x_0)\right]. \quad (30)$$

In the final term we obtain a double convolution of the form

$$\frac{2\epsilon}{x_0}\int_x^{x_0}\frac{dz}{1-\frac{z}{x_0}}\left[2zP_{qg}^{(0)}\left(\frac{z}{x_0}\right)\int_{x/z}^1 dz' r(z')\tilde{c}(x/zz') - x_0\int_{x/x_0}^1 dz' r(z')\tilde{c}(x/x_0z')\right]. \quad (31)$$

In principle this can be calculated, but it is convenient to make a change of variables and use y, z rather than z, z', where $y = zz'$. Doing this the first term in (31) becomes

$$\frac{2\epsilon}{x_0}\int_x^{x_0}\tilde{c}(x/y)\int_y^{x_0}\frac{dz}{z\left(1-\frac{z}{x_0}\right)}\,2zP_{qg}^{(0)}\left(\frac{z}{x_0}\right) r\left(\frac{y}{z}\right). \quad (32)$$

Changing variable in the second term gives

$$-\frac{2\epsilon}{x_0}\int_x^{x_0}\tilde{c}(x/y)\int_x^{x_0}\frac{dz}{x_0\left(1-\frac{z}{x_0}\right)}\,x_0 r\left(\frac{y}{x_0}\right). \quad (33)$$

The second integral is conveniently cut into two at $z = y$, producing

$$-\frac{2\epsilon}{x_0}\int_x^{x_0}\tilde{c}(x/y)\left(\int_y^{x_0}\frac{dz}{x_0\left(1-\frac{z}{x_0}\right)}\,x_0 r\left(\frac{y}{x_0}\right) + \int_x^{y}\frac{dz}{x_0\left(1-\frac{z}{x_0}\right)}\,x_0 r\left(\frac{y}{x_0}\right)\right). \quad (34)$$

Altogether (32) and first part of (34) gives a contribution of the form

$$\frac{2\epsilon}{x_0}\int_x^{x_0}\tilde{c}(x/y)\int_y^{x_0}\frac{dz}{\left(1-\frac{z}{x_0}\right)} \times\left[2P_{qg}^{(0)}\left(\frac{z}{x_0}\right) r\left(\frac{y}{z}\right) - r\left(\frac{y}{x_0}\right)\right], \quad (35)$$

while the second part of (34) gives a contribution of the form

$$\frac{2\epsilon}{x_0}\int_x^{x_0} dy\,\tilde{c}(x/y) r\left(\frac{y}{x_0}\right)\ln\left(\frac{1-y/x_0}{1-x/x_0}\right), \quad (36)$$

where the second integral over z has been performed explicitly. Equation (35) defines the '+' part while (36) effectively joins the local part (28).

Thus, we have all the ingredients to define $C_{\bar{\bar{s}}c}^{(0)VF}(\epsilon)\otimes\tilde{c}(Q^2)$. In order to obtain the complete LO expression for the generation of $\bar{\bar{s}}$ quarks in the VFNS we then have to add all these above ingredients to the LO FFNS expression frozen at $Q^2 = m_c^2$ as explained in [6]. Therefore

$$F_2^{LO,\bar{\bar{s}}}(x,Q^2) = \left(\frac{\alpha_S(m_c^2)}{2\pi}\right) C_{i,g}^{(1)FF}(1) \otimes \tilde{g}(m_c^2) + C_{\bar{\bar{s}}c}^{(0)VF}(\epsilon)\otimes\tilde{c}(Q^2), \quad (37)$$

for $Q^2 > m_c^2$, where

$$C_{\bar{\bar{s}}c}^{(0)VF}(\epsilon)\otimes\tilde{c}\left(Q^2\right) = -\int_x^{x_0} dz \left(\frac{dC_{i,g}^{(1)FF}(z,\epsilon)}{d\ln Q^2}\right)_{reg}\left(\frac{x}{z}\right)^2\frac{dc\left(x/z,Q^2\right)}{d\left(x/z\right)} + 2\int_x^{x_0} dx \left(\frac{dC_{i,g}^{(1)FF}(z,\epsilon)}{d\ln Q^2}\right)_{reg}\frac{x}{z}\,c\left(x/z,Q^2\right)$$

$$
\begin{aligned}
&-2\int_x^{x_0} dz\,\left(\frac{dC_{i,g}^{(1)FF}(z,\epsilon)}{d\ln Q^2}\right)_{reg}\int_{x/z}^1 dz'\,r(z')\,\frac{x}{zz'}\\
&\times c\left(x/zz',Q^2\right)-f_{\rm loc}\left(x,x_0\right)\left(\frac{x}{x_0}\right)^2\frac{dc\left(x/x_0,Q^2\right)}{d\left(x/x_0\right)}\\
&+2f_{\rm loc}\left(x,x_0\right)\,\frac{x}{x_0}\,c\left(x/x_0,Q^2\right)-\frac{2}{x_0}\int_x^{x_0}dz f_{\rm loc}\left(x,x_0\right)\\
&\times r\left(\frac{z}{x_0}\right)\,\frac{x}{z}\,c\left(x/z,Q^2\right)-2\frac{2\epsilon}{x_0}\int_x^{x_0}\,dy\,\tilde{c}\left(x/y\right)\\
&\times r\left(\frac{y}{x_0}\right)\ln\left(\frac{1-y/x_0}{1-x/x_0}\right)-\frac{2\epsilon}{x_0}\int_x^{x_0}\,\frac{dz}{1-\frac{z}{x_0}}\\
&\times\left[2zP_{qg}^{(0)}\left(\frac{z}{x_0}\right)\left(\frac{x}{z}\right)^2\frac{dc\left(x/z\right)}{d\left(x/z\right)}-x_0\left(\frac{x}{x_0}\right)^2\frac{dc\left(x/x_0\right)}{d\left(x/x_0\right)}\right]\\
&+2\frac{2\epsilon}{x_0}\int_x^{x_0}\,\frac{dz}{1-\frac{z}{x_0}}\left[2zP_{qg}^{(0)}\left(\frac{z}{x_0}\right)\tilde{c}\left(x/z\right)-x_0\tilde{c}\left(x/x_0\right)\right]\\
&-2\frac{2\epsilon}{x_0}\int_x^{x_0}\,\tilde{c}\left(x/y\right)\int_y^{x_0}\frac{dz}{\left(1-\frac{z}{x_0}\right)}\\
&\times\left[2P_{qg}^{(0)}\left(\frac{z}{x_0}\right)r\left(\frac{y}{z}\right)-r\left(\frac{y}{x_0}\right)\right]. \qquad (38)
\end{aligned}
$$

This expression then guarantees continuity of both the structure function and its derivative in $\ln Q^2$ as we switch from 3 to 4 flavours at $Q^2=m_c^2$.

Having completed the exercise for F_2 we can now do exactly the same thing for the phenomenologically interesting case of F_3. Once again we can first consider the contribution coming from the regular part of the FFNS coefficient function

$$(c)\qquad \frac{d}{d\ln Q^2}\left[C_{3,g,reg.}^{(1)FF}(\epsilon)\,\otimes\,\tilde{g}(Q^2)\right].$$

Differentiating the contribution to the structure function due to this we obtain

$$
\begin{aligned}
&\frac{d}{d\ln Q^2}\left[\int_x^{x_0}dz\,C_{3,g,reg.}^{(1)FF}\left(z,\epsilon\right)\,\tilde{g}\left(x/z\right)\right]\\
&=\epsilon x_0^2 C_{3,g,reg.}^{(1)FF}\left(x_0,\epsilon\right)\tilde{g}\left(x/x_0\right)\\
&\quad+\epsilon x_0^2\int_x^{x_0}\,dz\,\frac{d}{dx_0}\left[C_{3,g,reg.}^{(1)FF}\left(z,\epsilon\right)\right]\,\tilde{g}\left(x/z\right)\\
&=-\epsilon x_0^2\ln\left[x_0\left(1-x_0\right)\right]\tilde{g}\left(x/x_0\right)\\
&\quad+\epsilon x_0^2\int_x^{x_0}\,dz\,\frac{d}{dx_0}\left[C_{3,g,reg.}^{(1)FF}\left(z,\epsilon\right)\right]\,\tilde{g}\left(x/z\right), \qquad (39)
\end{aligned}
$$

and using (14) we get

$$
\begin{aligned}
&\frac{d}{dx_0}\left[C_{3,g,reg.}^{(1)FF}\left(z,\epsilon\right)\right]\\
&=\left[\frac{-2}{x_0\left(1-x_0\right)}\right]P_{qg}^{(0)}\left(\frac{z}{x_0}\right)+\frac{2z}{x_0^3}\left(x_0-2z\right)\\
&\quad\times\ln\left[\frac{x_0}{\left(1-x_0\right)z^2}\right]+\frac{4z}{x_0^3}\left(3z-2x_0\right). \qquad (40)
\end{aligned}
$$

We can then also consider the divergent part of the structure function

$$(d)\qquad \frac{d}{d\ln Q^2}\left[C_{3,g,dvgt.}^{(1)FF}(\epsilon)\,\otimes\,\tilde{g}(Q^2)\right].$$

The only difference compared to (b) above is an extra factor of x_0 so that

$$\frac{d\phi(z,\epsilon)}{d\ln Q^2}=\frac{4z}{x_0^3}(x_0-2z)$$

So (c) and (d) together give the following contributions:

$$
\begin{aligned}
\text{'local'term}: &\ \epsilon x_0^2\left[2\ln\left(1-\frac{x}{x_0}\right)-\ln\left(x_0\left(1-x_0\right)\right)\right]\\
&\tilde{g}\left(\frac{x}{x_0}\right) \qquad (41)\\
\text{`}+\text{'}\ \text{term}: &\ 2\epsilon\int_x^{x_0}\,\frac{dz}{1-\frac{z}{x_0}}\,\left[2zP_{qg}^{(0)}\left(\frac{z}{x_0}\right)\tilde{g}\left(x/z\right)\right.\\
&\left.-x_0\tilde{g}\left(\frac{z}{x_0}\right)\right] \qquad (42)\\
\text{'regular'term}: &\ \epsilon x_0^2\int_x^{x_0}\{\frac{d}{dx_0}C_{3,g,reg.}^{(1)}\left(z,\epsilon\right)|_{\text{(as given in (40))}}\\
&+\frac{4z}{x_0^3}\left(x_0-2z\right)\ln\left(1-\frac{z}{x_0}\right)\}\tilde{g}\left(x/z\right)\\
&\equiv\int_x^{x_0}dz\left(\frac{dC_{3,g}^{(1)}\left(z,\epsilon\right)}{d\ln Q^2}\right)_{reg}\tilde{g}\left(x/z\right). \qquad (43)
\end{aligned}
$$

As $Q^2/m_c^2\to\infty$, the only surviving term comes from the regular piece which $\to -2P_{qg}^{(0)}(z)$, and hence clearly using (8), $C_{3,\bar{\bar{s}}c}^{(0)VF}(\epsilon)\to -2z\delta(1-z)$ in this limit. As for F_2 we can use the above three contributions to construct the necessary $C_{3,\bar{\bar{s}}c}^{(0)VF}(\epsilon)\otimes\tilde{c}(Q^2)$ for the LO VFNS expression, i.e. we obtain the equivalent of (38) with the 'regular' part (27) replaced by (43), and the 'local' and '+' contributions being identical to those for F_2 up to a factor of x_0.

In practice, although it is convenient to talk about the production of c quarks and or $\bar{\bar{s}}$ quarks they are often produced together, and in order to define a physically relevant inclusive quantity we have to add the contributions we have considered for producing $\bar{\bar{s}}$ quarks, i.e. (5) in the FFNS and (37) in the VFNS and their analogues for F_3, to the expressions for charm production (4). Using these we have the contributions to the structure functions due to the production and/or conversion of heavy flavours. The relevant curves are shown in Figs. 1 and 2 for $F_2(x,Q^2)$ and $F_3(x,Q^2)$ respectively, where one can indeed see the continuity of both the structure functions and their derivatives, and the fact that they reduce to the correct limits at high and low Q^2[4]. (The small constant difference between the ZM-VFNS and the VFNS results at high Q^2 is due to the m_c^2-dependent first term on the rhs of (37), which as we argued in [6] it is correct to include.) Note that at LO we have to use parton densities evolved according to only the LO splitting functions.

[4] We use the preliminary set of partons described in [16]

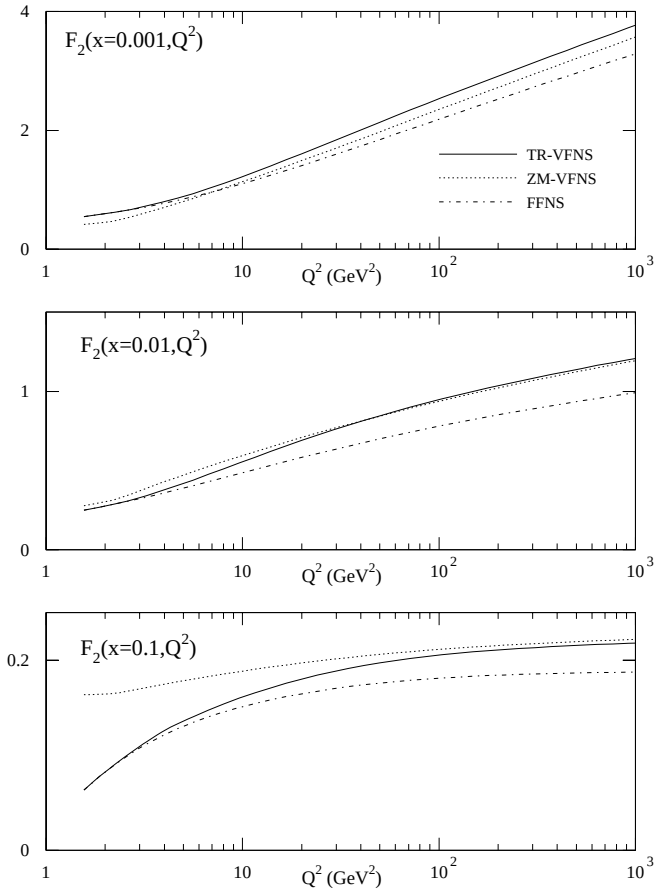

Fig. 1. Charm quark contribution to the structure functions, $F_2(x, Q^2)$ for $x = 0.1$, $x = 0.01$ and $x = 0.001$ calculated using our LO prescription, our input parton distributions evolved at LO and renormalization scale $\mu^2 = Q^2$. Also shown are the continuation of the LO FFNS expression and the ZM–VFNS expression both calculated using the same parton distributions and same choice of scale

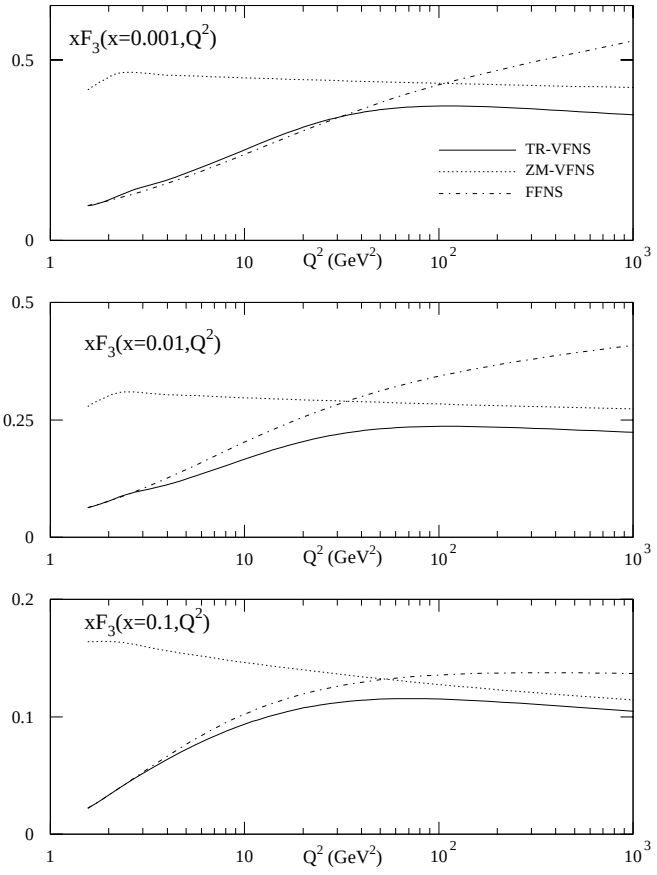

Fig. 2. Same as Fig. 1, but for $F_3(x, Q^2)$

3 The VFNS at NLO

We now consider the full range of NLO corrections to the charged-current structure functions. There is in principle a next-to-leading order correction to the production of charm quarks from $\tilde{s}$ quarks, i.e.

$$\left(\frac{\alpha_S}{4\pi}\right)\int_x^{x_0} dz\, C^{(1)}_{i,c\tilde{s}}(z,\epsilon)\,\bar{\tilde{s}}(x/z) \qquad (44)$$

where the $C^{(1)}_{i,c\tilde{s}}(z)$ are presented in [13]. These coefficient functions have no large logs in Q^2/m_c^2 and simply reduce to the correct massless expressions as $\epsilon \to 0$, and are the same in VFNS as in FFNS. However, the contribution from (44) is essentially negligible at all Q^2 and x. Hence, we use massless coefficient function for this process for simplicity.

There are then also other contributions at NLO. These are due to the coefficient functions $C^{(2)FF}_{i,g}(z,\epsilon)$, $C^{(2)FF,PS}_{i,\bar{\tilde{s}}q}(z,\epsilon)$ (where PS stands for pure singlet), $C^{(1)VF}_{i,\bar{\tilde{s}}c}(z,\epsilon)$ and $C^{(1)VF}_{i,g}(z,\epsilon)$. We will first consider the last of these, since this is the easiest to deal with.

The explicit form of (1) for $\mu^2 = Q^2$ at $\mathcal{O}(\alpha_s)$ is

$$c(z,Q^2) = \frac{\alpha_s}{2\pi}\,\ln\!\left(\frac{Q^2}{m_c^2}\right) P^0_{qg} \otimes g_{n_f=3}$$
$$g_{n_f=4}(z,Q^2) = g_{n_f=3}(z,Q^2) - \frac{\alpha_s}{6\pi}\,\ln\!\left(\frac{Q^2}{m_c^2}\right) g_{n_f=3}. \qquad (45)$$

Inserting the expressions for the matrix element $A^{cg}(z, \mu^2/m_c^2)$ into (3) gives the simple relation

$$C^{(1)\,FF}_{i,g}(z,\epsilon) = C^{(1)\,VF}_{i,g}(z,\epsilon) + C^{(0)\,VF}_{i,\bar{\tilde{s}}c}(\epsilon) \otimes P^0_{qg}\,\ln\!\left(\frac{1}{\epsilon}\right) \qquad (46)$$

connecting the $\mathcal{O}(\alpha_S)$ gluonic CF's in the FFNS and VFNS. Futhermore (11) allows the gluonic CF in the VFNS to be written as

$$C^{(1)\,VF}_{i,g}(\epsilon) \otimes \tilde{g}(Q^2)$$
$$= C^{(1)\,FF}_{i,g}(\epsilon) \otimes \tilde{g}(Q^2)$$
$$-\frac{d}{d\ln Q^2}(C^{(1)\,FF}_{i,g}(\epsilon) \otimes \tilde{g}(Q^2))\,\ln\!\left(\frac{1}{\epsilon}\right). \qquad (47)$$

Hence, it is a straightforward procedure to take the results of the previous section regarding the correct treat-

ment of $\frac{d}{d\ln Q^2}(C_{i,g}^{(1)\,FF}(\epsilon)\otimes\tilde{g}(Q^2))$, including the contributions from the endpoint of the integral in the convolution, to completely define $C_{i,g}^{(1)\,VF}(z,\epsilon)$. As with $C_{i,\bar{s}c}^{(0)\,VF}(z,\epsilon)$ there is a 'regular', 'local' and '+' contribution. Using the asymptotic limits for the FFNS coefficient functions in (16) and (17), along with the limits on their $\ln Q^2$ derivatives, presented in the previous section, we see that as $Q^2/m_c^2 \to \infty$, $C_{i,g}^{(1)\,VF}(z,\epsilon)$ do indeed tend to the correct asymptotic $\overline{\rm MS}$ limit.

In principle $C_{i,g}^{(2)FF}(z,\epsilon)$ and $C_{i,\bar{s}q}^{(2)FF,PS}(z,\epsilon)$ contribute at NLO. This is both in the FFNS expressions for $Q^2 < m_c^2$, and in the VFNS where the values frozen at $Q^2 = m_c^2$ are used to ensure continuity of the structure function. Unfortunately, unlike the neutral current case [15], neither of the contributions has been calculated yet, and as such we have no option but to leave them out completely. However, $C_{i,g}^{(2)FF}(z,\epsilon)$ also has a role to play in the definition of the NLO VFNS coefficient function $C_{i,\bar{s}c}^{(1)VF}(z,\epsilon)$, and it is not possible to simply claim ignorance and set this to zero since this would destroy the continuity of $\frac{dF_i(x,Q^2)}{d\ln Q^2}$ at NLO.

To see this we must consider the equation defining $C_{i,\bar{s}c}^{(1)VF}(z,\epsilon)$. This is analogous to the case for the neutral coupling discussed in Sect. 4 of the former of [6], and we have the definition

$$\frac{dC_{i,g}^{(2)FF}(z,\epsilon)}{d\ln Q^2} = C_{i,\bar{s}c}^{(1)VF}(\epsilon) \otimes \frac{dA_{cg}^{(1)}(\epsilon)}{d\ln Q^2} + C_{i,\bar{s}c}^{(0)VF}(\epsilon) \otimes \frac{dA_{cg}^{(2)}(\epsilon)}{d\ln Q^2} + \frac{1}{3\pi}\ln(1/\epsilon) C_{i,\bar{s}c}^{(0)VF}(\epsilon) \otimes P_{qg}^{(0)}, \quad (48)$$

where the last term comes about from the difference in the definition of the three and four flavour couplings. This expression would guarantee both the continuity of the $\ln Q^2$-derivative of the structure function at NLO (in the gluon sector), and the correct asymptotic expression for $C_{i,\bar{s}c}^{(1)VF}(z,\epsilon)$ - all terms containing a power of $\ln(1/\epsilon)$ being guaranteed to cancel. However, since we do not know the NLO FFNS coefficient function, we cannot therefore fully use the above equation. Nevertheless, simply putting $C_{i,\bar{s}c}^{(1)VF}(z,\epsilon)$ equal to its asymptotic value, which for the moment we consider to be in practice zero, is not consistent since this leads to the right-hand side of (48) being equal to

$$C_{i,\bar{s}c}^{(0)VF}(1) \otimes P_{qg}^{(1)} \quad (49)$$

at $Q^2 = m_c^2$, where we have used the expression for $\frac{dA_{cg}^{(2)}(\epsilon)}{d\ln Q^2}$ in (4.15) of the former of [6], whereas the left-hand side is zero. Thus, there is a mismatch between the lack of evolution at NLO for $Q^2 < m_c^2$, and from the NLO contribution to the evolution from the NLO quark-gluon splitting function for $Q^2 > m_c^2$ convoluted with the zeroth order coefficient function. This mismatch may be large, particularly at small x. In order to avoid this we have to invoke some ansatz for $C_{i,\bar{s}c}^{(1)VF}(\epsilon)$ so that the above contribution is cancelled. Using the fact that $\frac{dA_{cg}^{(1)}(z,\epsilon)}{d\ln Q^2} = P_{qg}^{(0)}(z)$, this results in the requirement

$$C_{i,\bar{s}c}^{(1)VF}(1) \otimes P_{qg}^{(0)} + C_{i,\bar{s}c}^{(0)VF}(1) \otimes P_{qg}^{(1)} = 0. \quad (50)$$

The minimal way in which to satisfy this, and to ensure that $C_{i,\bar{s}c}^{(1)VF}(\epsilon) \to 0$ as $\epsilon \to 0$, is to demand that

$$C_{i,\bar{s}c}^{(1)VF}(\epsilon) \otimes P_{qg}^{(0)} = -\epsilon\, C_{i,\bar{s}c}^{(0)VF}(\epsilon) \otimes P_{qg}^{(1)}. \quad (51)$$

In principle this is the definition we use for $C_{i,\bar{s}}^{(1)VF}(z,\epsilon)$, but this would be extremely complicated to implement in practice. Since the coefficient function is always convoluted with a parton distribution, and is based on the known $C_{i,\bar{s}c}^{(0)VF}(z,\epsilon)$ we find an appropriate modification of $C_{i,\bar{s}}^{(0)VF}(z,\epsilon)$ necessary to account for the effect of the NLO coefficient function. We find that assuming that the parton distribution takes roughly the form $(1-x)^8x^{-0.3}$ then we can model the action of this NLO coefficient function by replacing all terms of the form

$$\frac{dC_{i,g}^{(1)FF}(z,\epsilon)}{d\ln Q^2} \otimes \tilde{f}(x/z,Q^2) \quad (52)$$

occurring in (38) by

$$\frac{dC_{i,g}^{(1)FF}(z,\epsilon)}{d\ln Q^2} \otimes \left(1 - \epsilon\frac{38\alpha_S(Q^2)}{4\pi}(\ln(4+(x/z)^{-0.25}) - \ln(4) - 2(x/z)\right)\tilde{f}(x/z,Q^2), \quad (53)$$

with analogous modifications for the 'local' and '+' contributions. It can be checked explicitly that this does indeed represent the exact expression (51) very accurately. The main effect is an opposite sign correction to the LO result which increases in magnitude as one goes to smaller x. Examining (51) one sees that $C_{i,\bar{s}c}^{(1)VF}$ depends on $P_{qg}^{(1)}/P_{qg}^{(0)}$ (where the division is really only illustrative since convolutions are involved), and since $P_{qg}^{(1)}$ grows much more quickly at small x than $P_{qg}^{(0)}$ this effect is fully expected. Finally, we also add a contribution of $(1-\epsilon)C_{i,\bar{s}c}^{(1)ZM-VF}$ in order to obtain the correct asymptotic limit, though the contribution due to this is tiny at small Q^2.

This completes our definition of the VFNS for charged current scattering at NLO. Unfortunately a complete definition will have to await the calculation of the unknown NLO FFNS coefficient functions, but we are confident that this will lead to only small corrections, mainly for $Q^2 \lesssim m_c^2$. In Figs. 3 and 4 we plot the contributions to $F_2(x,Q^2)$ and $F_3(x,Q^2)$ respectively due to the production and/or conversion of charm quarks at NLO[5]. Once

[5] Again using partons from [16]

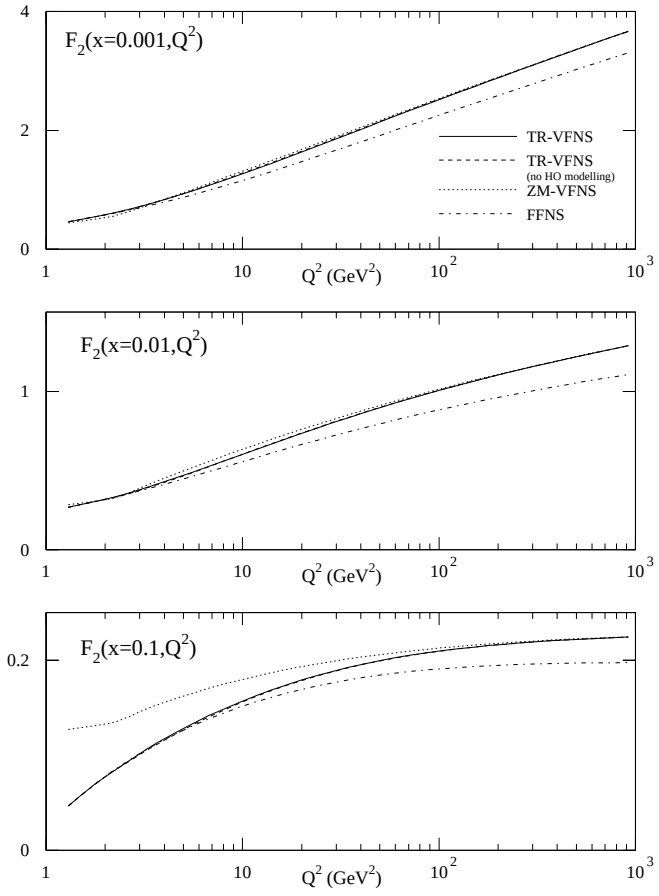

Fig. 3. Charm quark contribution to the structure functions, $F_2(x, Q^2)$ for $x = 0.1$, $x = 0.01$ and $x = 0.001$ calculated using our NLO prescription, our input parton distributions evolved at NLO and renormalization scale $\mu^2 = Q^2$. Also shown are the continuation of the FFNS expression with LO coefficient functions (those at NLO being unavailable) and the NLO ZM–VFNS expression both calculated using the same parton distributions and same choice of scale. Also shown for comparison is the VFNS result when $C_{2,\bar{s}}^{(1)VF}$ is set equal to zero

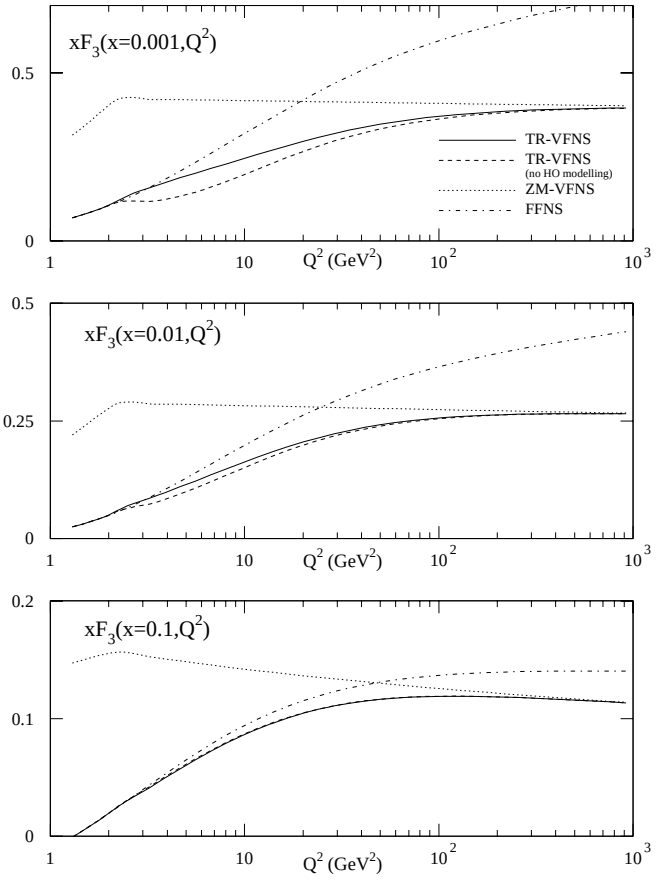

Fig. 4. Same as Fig. 3, but for $F_3(x, Q^2)$

again one can see the continuity of the structure functions and their derivatives, and the correct asymptotic behaviour[6] (we plot the FFNS result obtained from the LO coefficient functions since those at NLO are not known). This time we use partons evolved at NLO. For comparison we also plot the NLO structure functions with $C_{i,\bar{s}c}^{(1)VF}$ set equal to zero. One can see that at small x this does indeed lead to a clear discontinuity in the derivative in the structure function at the transition point $Q^2 = m_c^2$, particularly as one goes to smaller x. This is, however, far more clear for $F_3(x, Q^2)$ where we are calculating roughly the difference between the strange and charm quark distributions and the discrepancy in the evolution affecting one shows up much more obviously than for $F_2(x, Q^2)$, which is roughly the sum of the two quark distributions.

[6] This time the high Q^2 limits of the VFNS and the ZM-VFNS are identical since the constant difference would depend on the unknown NLO FFNS coefficient functions

One can extend the treatment to higher orders in principle following the general outline provided in the former of [6]. As mentioned in the introduction, at NNLO and beyond there is a complication in so much that particular flavours may be generated in the final state due to the cutting of quark loops produced away from the interaction vertex with the external gauge boson. This highlights the experimental ambiguity in defining heavy flavour structure functions and in principle one needs define some kinematic cut on such quarks to decide whether they are included or not. This issue is treated in [5] for the neutral current case, though in practice the effect is extremely small. Since for charged currents we do not even have a complete definition of the VFNS or FFNS at NLO this issue is not particularly pressing at the moment.

4 $\Delta xF_3(x, Q^2)$

As we can see from Figs. 1–4 our VFNS works well, ensuring smoothness and the correct limits. We can repeat exactly the same procedure for the process $W^- \to \bar{c}, \tilde{s}$, and this then allows the calculation of combined neutrino and antineutrino cross-sections, as measured by CCFR, and also the currently interesting quantity $\Delta xF_3 = xF_3^{\nu N} - xF_3^{\bar{\nu} N}$ [10] where N represents an isoscalar target. We present our results for ΔxF_3 in Fig. 5, for the range of x

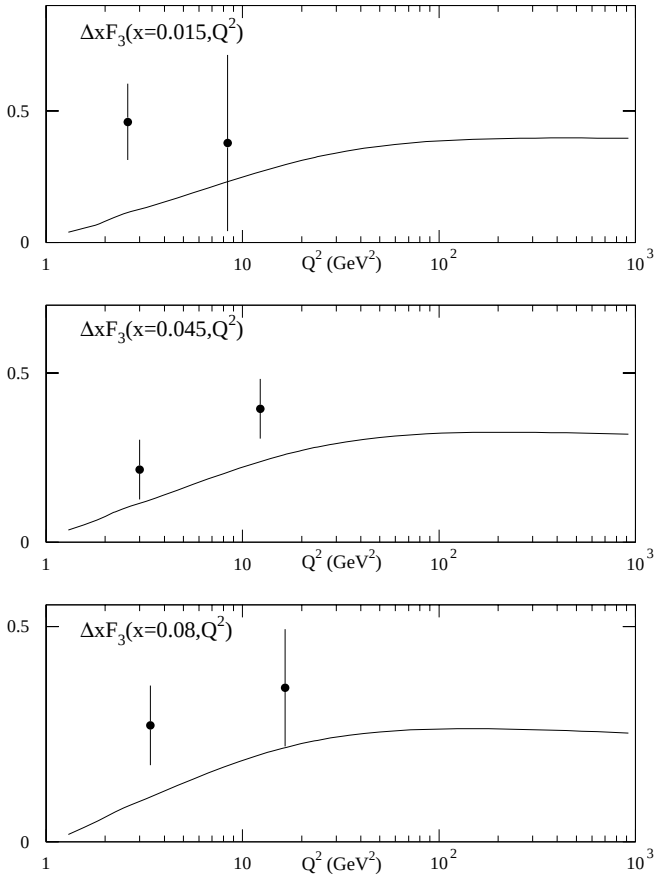

Fig. 5. The NLO prediction for $\Delta xF_3(x,Q^2)$ using our VFNS prescription, along with the data measured by CCFR [10]. The prediction has been corrected for heavy target effects using [20]

relevant for the CCFR experiment ($x > 0.01$). The curves are extremely similar to those for $xF_3(x,Q^2)$ (with a factor of two), and would show the same type of kink at low x if $C^{(1)VF}_{3,\bar{\tilde{s}}c,(\bar{c}\tilde{s})}(z,\epsilon)$ were set equal to its asymptotic value or to zero. We also present the data on ΔxF_3 measured by CCFR [10], and note that our predictions lie considerably beneath the measurements. Possible reasons for this are considered in [11]. Since the data is at quite low Q^2 it is clear that the FFNS would lead to very similar predictions. By comparison with Fig. 4 one can see that there would be a very slight improvement for the higher Q^2 points, but only due to missing contributions correctly accounted for in the VFNS. Similarly the ZM-VFNS would actually compare to the data fairly well, but is simply incorrect at such low Q^2.

5 Summary

In this paper we have explicitly constructed a VFNS for the production and conversion of heavy flavours for the case of charged currents. We have demonstrated that the predicted structure function is very well described over a wide range of x and Q^2 - having the correct asymptotic limits for low Q^2 and for $Q^2 \to \infty$. We note that a VFNS is particularly important in this case. For $Q^2 \sim m_c^2$ there is no reason why the ZM-VFNS should be a particularly good approximation to the correct structure function since it is missing essential information on the kinematics. Indeed, it is not that successful in the neutral current case, often leading to a negative F_2^c for low Q^2 and being much too high for F_L^c as seen in e.g. the first of [6]. It has, however, been argued, e.g. [17], that the FFNS is sufficient even up to $Q^2 >> m_c^2$, and for the neutral current F_2 it seems arguable that this is correct (particularly if the renormalization/factorization scale is chosen judiciously). However, it was demonstrated in [18] that particularly for the case of F_3, which is best measured in neutrino scattering, this is no longer true, and at high Q^2 the FFNS expansion is very slow to converge towards a resummed VFNS result and changes considerably from order to order. Hence, in this case the FFNS is clearly unreliable at high Q^2 and a VFNS is needed.

Our particular scheme is built upon two basic ideas - incorporation of the correct kinematic behaviour into each coefficient function by imposition of the continuity of $(dF(x,Q^2)/d\ln Q^2)$ across the transition point $Q^2 = m_c^2$, and a correct ordering of the expansion in α_S, so that a well-defined expansion scheme is used in each limit and in between. However, these two ideas are linked by the complete definition. In the case of charged currents the former no longer appears to be such a direct benefit as for the neutral current case, because even the lowest order boson-gluon fusion diagram needs a collinear subtraction due to the final state light quark, and thus the finite part is not a true parton cross-section. This means that unlike for the quark-antiquark production in the neutral current case the coefficient function does not vanish at threshold, and is even divergent. This leads both to technical difficulty, with our coefficient functions containing '+' distributions, and to there being a less direct link between the coefficient functions and the physics. As such, superficially there seems to be no advantage compared to other VFNSs. Nevertheless, the ordering still remains an advantage. Not only is it theoretically correct, combining renormalization/factorization scheme independence up to higher orders with continuity of structure functions, but it has a clear phenomenological benefit. This becomes particularly clear at NLO, where the ordering and the continuity of the $\ln Q^2$-derivative of the structure function impose conditions on $C^{(1)VF}_{i,\bar{\tilde{s}}c}$ even in the absence of the NLO FFNS coefficient functions, forcing smoothness by relating this NLO coefficient function to the NLO evolution. Other schemes, e.g. [3,5], do not have the same type of definition of $C^{(1)VF}_{i,\bar{\tilde{s}}c}$, i.e. do not relate it to $P^{(1)}_{qg}$ and would, we believe, have similar behaviour to our curves with this coefficient function set equal to zero if the scale $\mu^2 = Q^2$ were used. This unphysical behaviour would, however, be reduced if arguably more physical scales, such as $\mu^2 = Q^2 + m_c^2$ were used.

Along with this paper we will make available new code for calculating the heavy flavour contribution to charged

currents[7]. This contains various changes and corrections compared to the previous version. In particular we no longer use the coefficient function in [19] for calculation of $F_2(x, Q^2)$ in the charged current case, since this seems to be incompatible with those in [13] and [14], and we choose to believe these since [13] has been extensively cross-checked[8]. This change in coefficient functions leads to a significant reduction in $F_2(x, Q^2)$ at low Q^2, though the difference disappear at high Q^2. Thus, we now have a complete, explicit prescription for the production of charged current structure functions including heavy flavour effects which may be used along with LO or NLO $\overline{\rm MS}$ partons distributions. We hope this will prove useful to the community.

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[7] The FORTRAN code for this prescription can be obtained from `http://durpdg.dur.ac.uk/HEPDATA/PDF`

[8] We would like to thank Stefan Kretzer for details on this